

AS-Level Pure Mathematics Diagnostic Test

Baseline Assessment for Year 13 Entry

Time: 1 Hour | Total Marks: 76

Instructions

- Answer all questions.
 - Show sufficient working to make your methods clear. Answers without working may not gain full credit.
 - Calculators may be used, except where otherwise stated.
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1. [From Algebra Part 1] Show that

$$\frac{2}{\sqrt{12} - \sqrt{8}}$$

can be written in the form $\sqrt{a} + \sqrt{b}$, where a and b are integers. [5]

2. [From Quadratics]

- (a) Express $x^2 + 2x + 3$ in the form $(x + a)^2 + b$, where a and b are constants to be found. [2]
- (b) Sketch the graph of $y = x^2 + 2x + 3$, indicating clearly the coordinates of any intersections with the coordinate axes. [3]
- (c) Find the value of the discriminant of $x^2 + 2x + 3$. Explain how the sign of the discriminant relates to your sketch in part (b). [2]
- (d) The equation $x^2 + kx + 3 = 0$, where k is a constant, has no real roots. Find the set of possible values of k , giving your answer in surd form. [4]

3. [From Algebra Part 2] Solve the simultaneous equations:

$$\begin{aligned}x + y &= 2 \\ 4y^2 - x^2 &= 11\end{aligned}$$

[7]

4. [From Binomial Expansion]

- (a) Find the first 3 terms, in ascending powers of x , of the binomial expansion of $(3 + bx)^5$ where b is a non-zero constant. Give each term in its simplest form. [4]
- (b) Given that, in this expansion, the coefficient of x^2 is twice the coefficient of x , find the value of b . [2]

5. **[From Coordinate Geometry]** The circle C has centre $P(7, 8)$ and passes through the point $Q(10, 13)$.

(a) Find the length PQ , giving your answer as an exact value. [2]

(b) Hence write down an equation for C . [2]

(c) The line l is a tangent to C at the point Q . Find an equation for l , giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. [4]

6. **[From Exponentials and Logarithms]** Find the values of x such that:

$$2 \log_3 x - \log_3(x - 2) = 2$$

[5]

7. **[From Differentiation 1]** A manufacturer produces tablets in the shape of a solid circular cylinder with base radius x mm and height h mm. Given that the volume of each tablet is 60 mm^3 :

(a) Express h in terms of x . [1]

(b) Show that the surface area, $A \text{ mm}^2$, of a tablet is given by $A = 2\pi x^2 + \frac{120}{x}$. [3]

(c) Use calculus to find the value of x for which A is a minimum. [5]

(d) Calculate the minimum value of A , giving your answer to the nearest integer. [2]

(e) Show that this value of A is a minimum. [2]

8. **[From Integration]**

(a) Find $\int (3x - x^{\frac{3}{2}}) dx$. [3]

(b) Hence find the area of the finite region S , bounded by the x -axis and the curve $y = 3x - x^{\frac{3}{2}}$. [3]

9. **[From Trigonometry]**

(a) Show that the equation $\sin \theta \tan \theta = 3 \cos \theta + 2$ can be written in the form $4 \cos^2 \theta + 2 \cos \theta - 1 = 0$. [3]

(b) Hence solve, for $0 \leq \theta < 360^\circ$, the equation $\sin \theta \tan \theta = 3 \cos \theta + 2$, showing each stage of your working. [5]

10. **[From Graph Transformations]** The curve C has equation $y = f(x)$, where $f(x) = (x - 1)(x - 3)^2$.

(a) Sketch the curve $y = f(x)$, showing clearly the coordinates of any points where the curve meets or crosses the coordinate axes. [3]

(b) On separate axes, sketch the curve with equation:

(i) $y = f(x + 2)$ [2]

(ii) $y = 2f(x)$ [2]

On each sketch, state the coordinates of any points where the curve meets or crosses the coordinate axes.

END OF TEST